

# Computer Vision

## Day 4

2017, Tanta University

**Eng.**Sara Hussien

# Hough Transform

# An edge is not a line...



How can we detect *lines*?

# Edges vs Boundaries

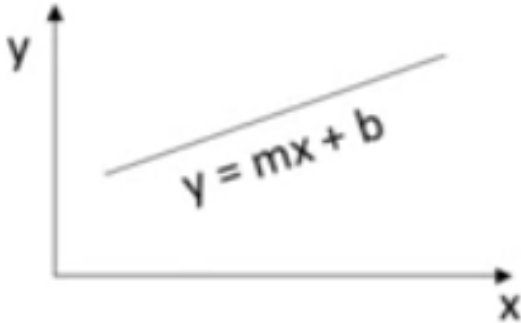
- Edges
  - Local intensity discontinuities
  - Points
  - Not dependent on models
- Boundaries
  - Composed of many points
  - May be dependent on model

# Finding lines in an image

- Option 1:
  - Search for the line at every possible position/orientation
  - What is the cost of this operation?
- Option 2:
  - Use a voting scheme: Hough transform

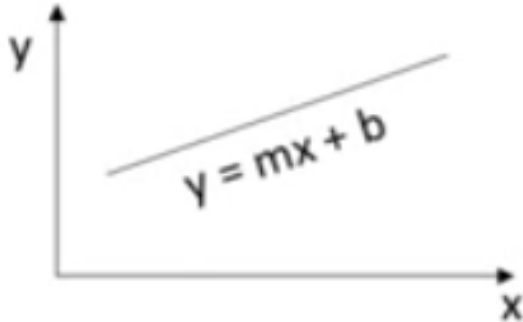
# Finding lines via Hough Transform

- A line has a two parameters (m,b)



# Finding lines via Hough Transform

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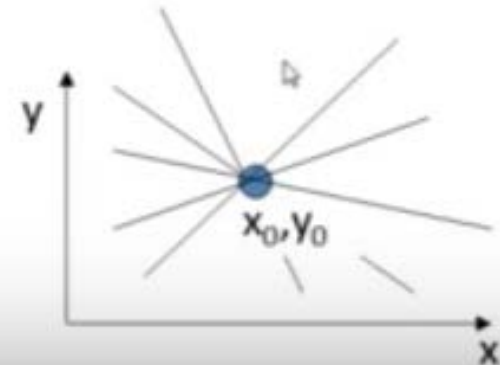
- Given a point  $(x_0, y_0)$ , the lines that could pass through this points are all (m,b) satisfying

$$y_0 = m x_0 + b$$

- Or

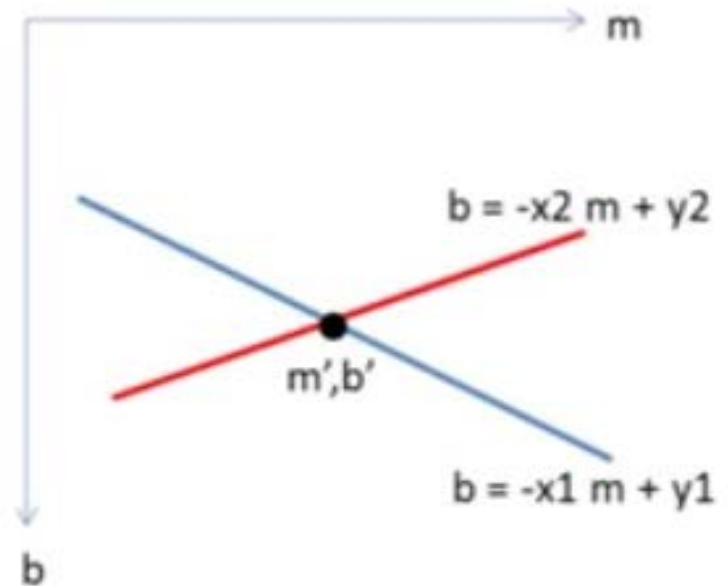
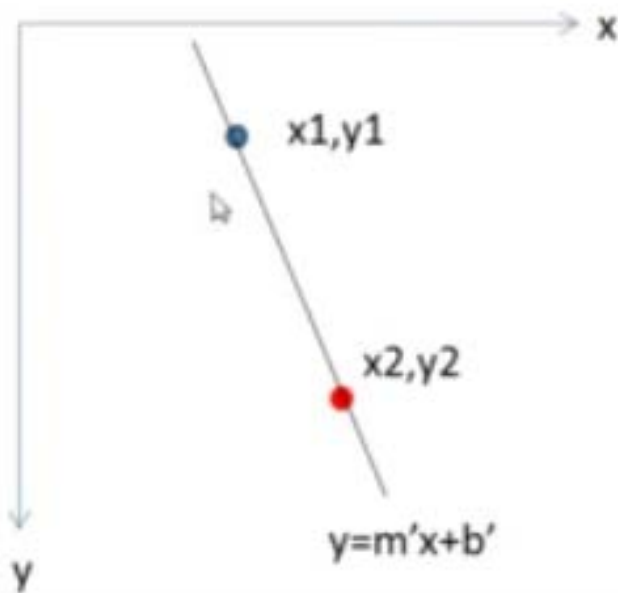
$$b = -x_0 m + y_0$$

The equation  $b = -x_0 m + y_0$  is a line in (m,b) space



# Hough Transform (Continued)

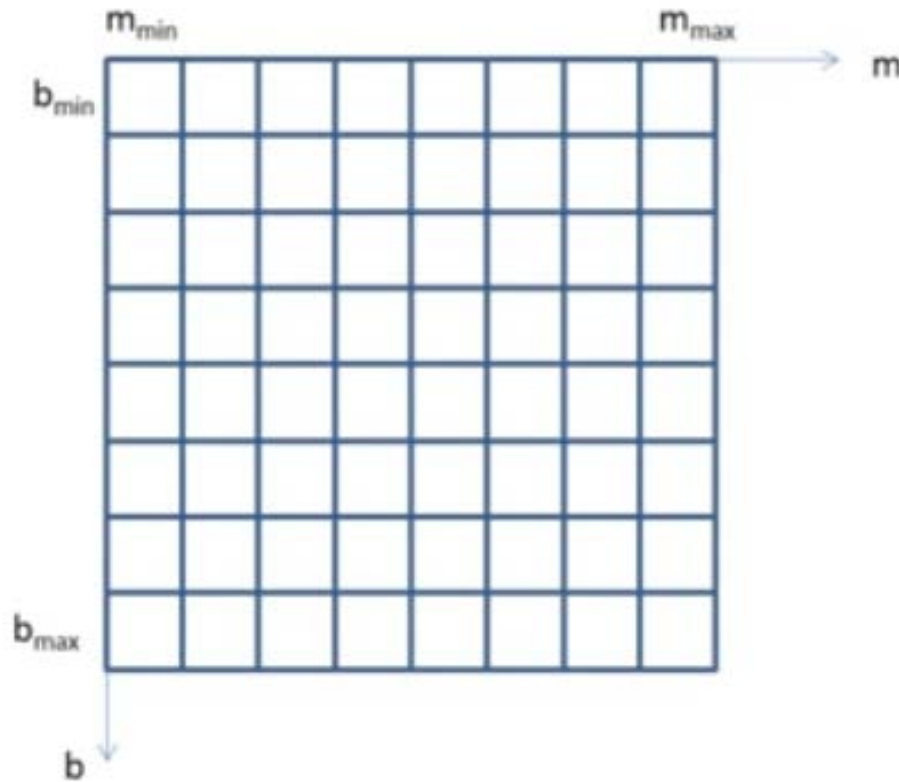
- All points on a line in image space, yield lines in parameter space which intersect at a common point
  - this point is the  $(m,b)$  of the line in image space.





# Hough Transform Algorithm

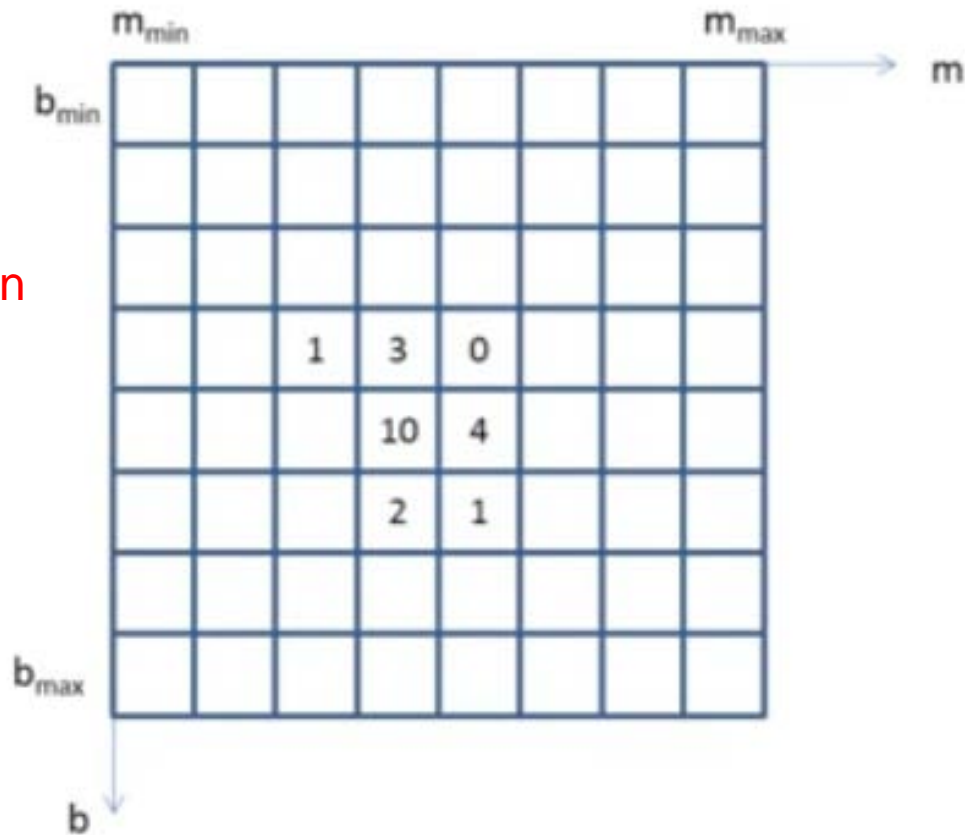
- Initialize an accumulator array  $A(m,b)$  to zero





# Hough Transform Algorithm

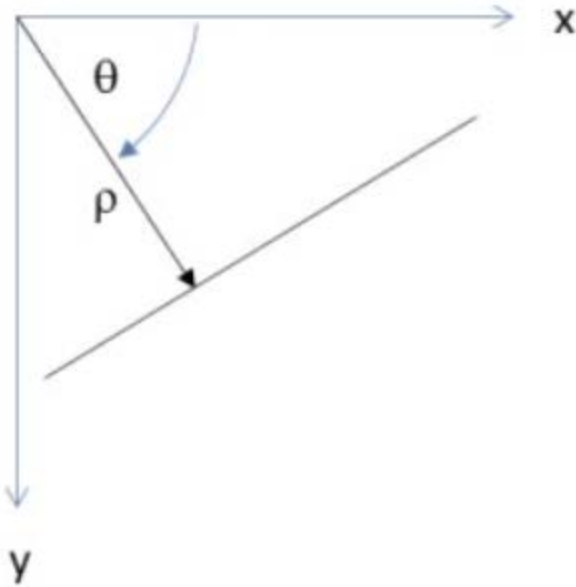
- Initialize an accumulator array  $A(m,b)$  to zero
- For each edge element  $(x,y)$ , increment all cells that satisfy  $b = -x m + y$



“This representation fails in case of vertical lines”

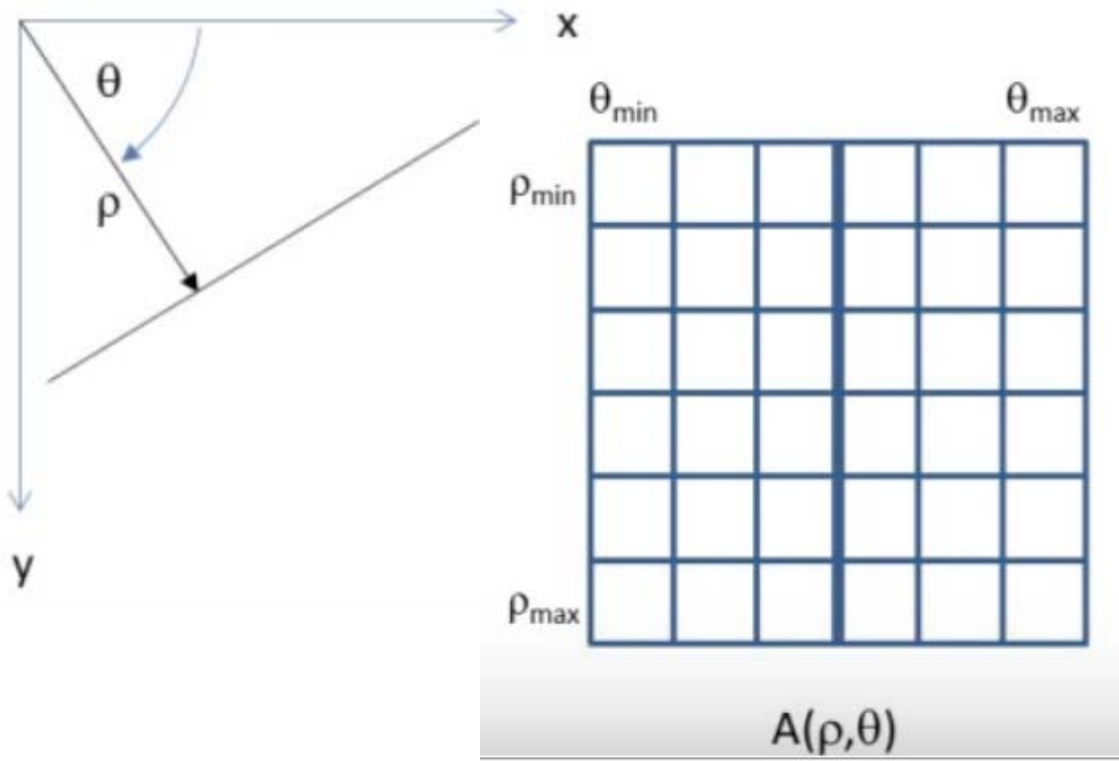
# Polar Coordinate Representation of Line

- $\rho = x \cos \theta + y \sin \theta$ 
  - $d$  is the perpendicular distance from the line to the origin
  - $\theta$  is the angle this perpendicular makes with the  $x$  axis
- Avoids infinite slope

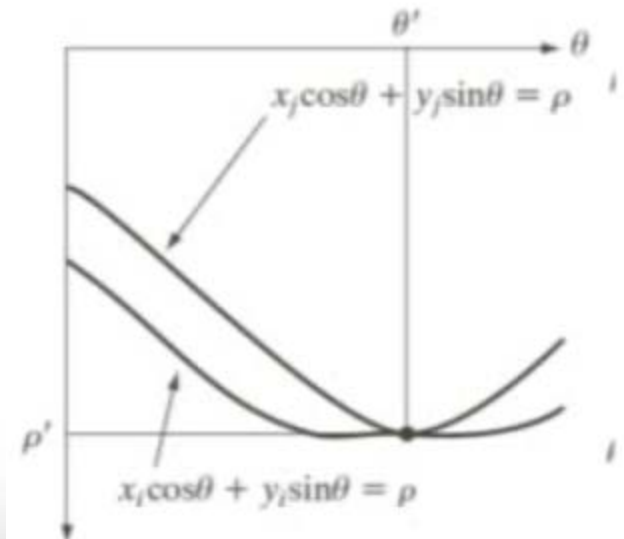


# Polar Coordinate Representation of Line

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The parameter space transform of a point is a sinusoidal curve



# Basic Hough transform algorithm

1. Quantize the Hough Transform space: identify the maximum and minimum values of  $\rho$  and  $\theta$
2. Generate an accumulator array  $H[d, \theta]$ ; set all values to zero
3. for each edge point  $I[x,y]$  in the image  
    for  $\theta = 0$  to  $180$   
        
$$d = x\cos\theta + y\sin\theta$$
  
        
$$H[d, \theta] += 1$$
4. Find the value(s) of  $(d, \theta)$  where  $H[d, \theta]$  is maximum
5. Calculate the equation of the line  $d = x\cos\theta + y\sin\theta$

# Extensions

- Extension 1 : Use the image gradient
  - To reduce the computational load use Gradient information

In step 2 : for each edge point  $I[x,y]$  in the image

compute unique  $(d, \theta)$  based on image gradient at  $(x,y)$

$H[d, \theta] += 1$

- To reduce the effect of noise more than one element (elements in a neighborhood) in the accumulator array are increased

# Extensions

- Extension 2 :
  - The same procedure can be used with circles, squares, or any other shape



# Hough Transform for Detection of Circles

- The parametric equation of the circle can be written as

$$(x - a)^2 + (y - b)^2 = r^2$$

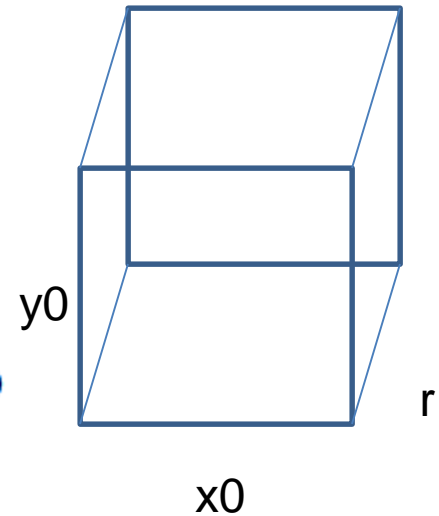
- The equation has three parameters – a, b, r
- The curve obtained in the Hough Transform space for each edge point will be a right circular cone
- Point of intersection of the cones gives the parameters a, b, r

# Hough Transform for Detection of Circles

Example- circles  $(x_0, y_0, r)$

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

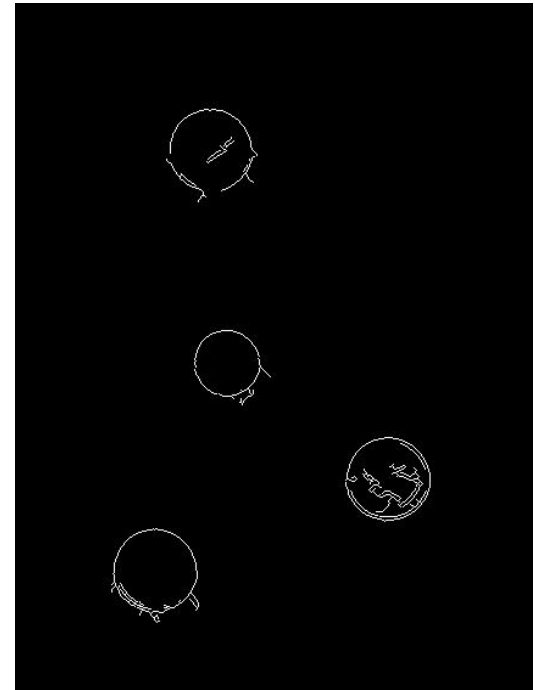
```
for all x
  for all y
    if E(x,y)
      for all x0
        for all y0
          r=root((x-x0)^2+(y-y0)^2)
          increment H at (x0,y0,r)
        end
      end
    end
  end
end
```



# Detection of circle by Hough Transform - example

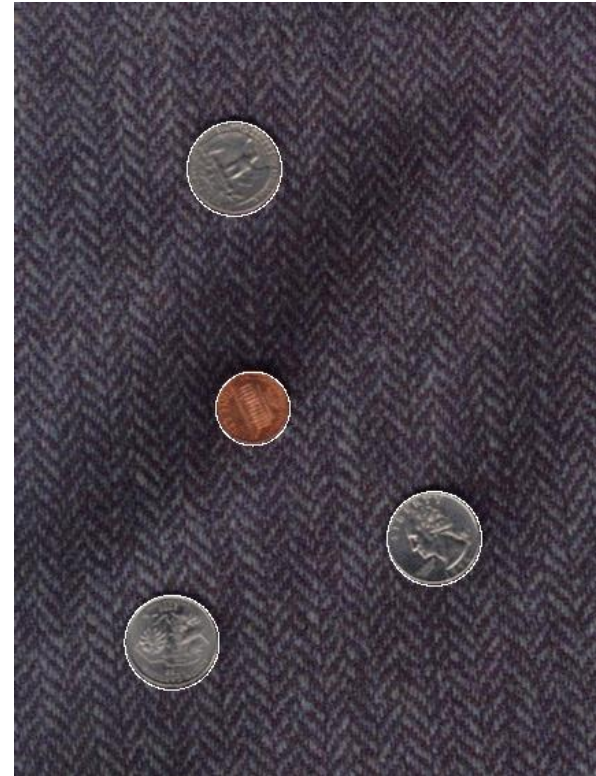
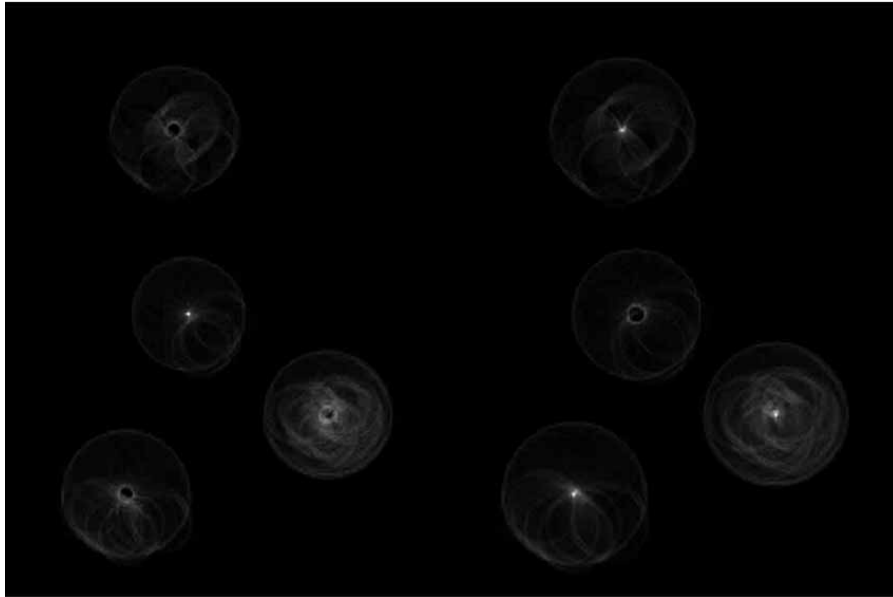


Original Image



Circles detected by Canny Edge Detector

# Detection of circle by Hough Transform - contd



Hough Transform of the edge detected image

Detected Circles

